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D-Moduli Stabilization*

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Abstract

The matter sector of four-dimensional effective supergravity models obtained from the weakly coupled heterotic string contains many moduli. In particular, flat directions of the D-term part of the scalar potential in the presence of an anomalous $U(1)$ give rise to massless chiral multiplets which have been referred to elsewhere as *D-moduli.* The stabilization of these moduli is necessary for the determination of the large vacuum expectation values of complex scalar fields induced by the corresponding Fayet-Illiopoulos term. This stabilization is of phenomenological importance since these background values determine the effective theory below the scale of the anomalous $U(1)$ symmetry breaking. In some simple models we illustrate the stabilization of these moduli due to the nonperturbative dynamics associated with gaugino condensation in a hidden sector. We find that background field configurations which are stable above the condensation scale no longer represent global minima once dynamical supersymmetry breaking occurs. The implications for low energy models based on promising "flat" directions are discussed.

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1 Introduction

Cancellation of the trace anomaly associated with an anomalous $U(1)_X$ by the GS mechanism [1] leads to an FI term [2] for the D-term of $U(1)_X$:

$$D_X = \sum_i K_i q_i^X \phi^i + \xi, \qquad \xi = \frac{g_H^2 \operatorname{tr} Q_X}{192\pi^2} m_P^2.$$

$K_i = \partial K/\partial\phi^i$ is the field derivative of the Kähler potential, g_H is the unified coupling at the string scale Λ_H, the charge of the scalar field ϕ^i is given by q_i^X and $m_P = 1/\sqrt{8\pi G} = 2.44 \times 10^{18}$ GeV is the reduced Planck mass.

Large matter field vacuum expectation values (*vevs*) are generic in semi-realistic string-inspired models because anomalous $U(1)_X$'s are generic. In [3] it was shown that the presence of a $U(1)_X$ factor in the gauge group G is generic for semi-realistic Z_3 orbifold models. For the class of standard-like orbifolds studied there, only 7 of 175 models *did not* have a $U(1)_X$. In the semi-realistic free fermionic models [4] a $U(1)_X$ is also generic.

The n fields ϕ^i which acquire nonvanishing vevs $\langle\phi^i\rangle \equiv v^i$ will be referred to here as *X-Higgs* fields. For canonical $K = \sum_i |\phi^i|^2$, we have 1 constraint:

$$\langle D_X \rangle = \sum_i q_i^X |v_i|^2 + \xi. \tag{1.1}$$

These vevs are generically complex. Indeed, (1.1) is completely "phase-blind." In addition, $\langle D_X\rangle = 0$ only constrains the magnitudes of the vevs to take values on a n-dimensional hyperboloid. The moduli which parameterize the unconstrained complex phases and the location on the n-dimensional hyperboloid are flat directions of the D-term part of the scalar potential and are pseudo-Goldstone bosons which were termed *D-moduli* in a previous work done in collaboration with Mary K. Gaillard (MKG) [5]. The D-moduli correspond to the $U(r, n-r)$ invariance of $\langle D_X\rangle = 0$, where r is the number of fields with $q_i^X > 0$. Only one of these D-moduli chiral multiplets gets "eaten" by the $U(1)_X$ vector multiplet when it gets massive.

This vacuum degeneracy is a generic problem in supersymmetric models [6]. Typically one chooses a flat direction which gives rise to "good" phenomenology. However this is arbitrary and may not be consistent with the dynamics employed to break supersymmetry. An effective theory of supersymmetry breaking can potentially lift most vacuum degeneracy. Accounting for effects of dynamical supersymmetry breaking, *arbitrariness in the phenomenology due to X-Higgs vevs can be removed.*

Here, we assume supersymmetry breaking via gaugino condensation in a hidden sector. Improvements in our understanding of strongly coupled super-Yang-Mills (embedded into string-derived effective supergravity) increase the reliability of this approach toward solving vacuum selection problems. It is already known that different string embeddings are related to each other by X-Higgs vevs. For example, in the work of Aldazabal et al. [7] $k = 1$ constructions and $k = 2$ constructions are related at special values of X-Higgs vevs. Thus an effective field theory approach to dynamical vacuum selection can make modest progress in the vacuum selection problem of string theory.

2 Scope

Efforts are underway in work with MKG to stabilize these moduli in semi-realistic models by including various terms in V (intentionally) neglected in our earlier work [5]. MKG has found ways to: consistently fix to unitary gauge (not an easy task with dynamical string moduli); preserve manifest target-space modular invariance; effectively include tree-level exchange of heavy multiplets in the general case of many fields and $U(1)$'s. The machinery is forthcoming [8].

Here I report only on stabilization in some rather simple "toy" models. I do not account for tree-level exchange of the heavy fields. I do not include compactification moduli or target space modular invariance. I oversimplify so as to isolate the issue of D-moduli stabilization. Fixing to unitary gauge is simple when string moduli are treated as constant background fields.

3 Background

In [5], the scalar potential V for SUGRA with a $U(1)_X$ was studied for vacuum configuations satisfying

$$\langle V \rangle = \langle \partial V / \partial \phi^i \rangle = 0. \tag{3.1}$$

Supersymmetry breaking was characterized by

$$\frac{1}{m_P^4}\langle |W|^2 \rangle = |\delta|^2, \qquad \frac{1}{m_P^2}\langle K_{i\bar{j}} F^i \bar{F}^{\bar{j}} \rangle = \alpha |\delta|^2 e^{\langle K \rangle / m_P^2}, \qquad \alpha \sim \mathcal{O}(1).$$

According to expectations, it was found that (3.1) together with a reasonable supersymmetry breaking scale $|\delta| \sim 1$ TeV requires

$$\langle D_X \rangle \sim |\delta|^2 \ll |\xi|.$$

We found that in the stable vacuum only fields with the minimum charge $\min\{q_i\}$ can get vevs. One combination of fields from this set gets eaten by the $U(1)_X$ multiplet while the remainder are massless *after* supersymmetry breaking! For canonical Kähler potential

$$\langle D_X \rangle = \sum_i q_i^X |v_i|^2 + \xi, \quad q_i^X \sim 1 \quad \Rightarrow \quad |v^i| \sim \sqrt{|\xi|}.$$

Research in progress with MKG finds that these order of magnitude relations hold in cases more complicated than those studied in our earlier work. This is also in agreement with work of Barreiro et al. [9].

Based on $|v^i| \sim \sqrt{|\xi|}$, the $U(1)_X$ gauge symmetry breaking scale Λ_X may be defined as

$$\Lambda_X = \sqrt{|\xi|}.$$

For the class of models studied in [3] it was found that for the 168 of 175 cases where $\xi \neq 0$,

$$\frac{g_H}{8.00} \le \frac{\Lambda_X}{m_P} \le \frac{g_H}{4.63} = \frac{\Lambda_H}{m_P}.$$

where $\Lambda_H \approx 0.216 \times g_H m_P$ is the approximate string scale obtained by Kaplunovsky [10]. With $g_H \sim 1$ we have that $\Lambda_X \sim 0.1 \times m_P$ is a generic prediction. The result of this is that nonrenormalizable operators should contribute *significantly* to the (effective) Yukawa couplings of the lighter quarks, since they are only down by $(\Lambda_X/m_P)^n \sim 10^{-n}, n > 0$. Operators with $1 \le n \le 4$ would typically be present.[1] Given $\lambda_{u,d}/\lambda_t \sim 10^{-5}$ after running to the high scale, it is difficult to believe that nonrenomalizable operators would not play a role, generically speaking. This serves as an example of how stabilization of the D-moduli is a crucial ingredient in predicting low-energy physics.

4 Effective scalar potential

This is a modification of the linear multiplet (L) toy model considered in earlier work with MKG [5]. The desire is to lift vacuum degeneracy by coupling D-moduli to matter condensates of the hidden sector condensing group G_C. Such couplings are expected[2] from the mixed trace anomaly matching condition $\operatorname{tr} T^a T^a Q_X \neq 0$, where T^a is a generator of G_C. We have

$$\begin{aligned} K &= k(L) + G(A, B, \Phi, \bar{A}, \bar{B}, \bar{\Phi}), \qquad k(L) = \ln L + g(L), \\ G &= \sum_i |A_i|^2 + \sum_i |B_i|^2 + \sum_i |\Phi_i|^2. \end{aligned}$$

The chiral superfields Φ_i are supposed to be the X-Higgses. We denote the scalar components ϕ_i and the corresponding vevs $v_i = \langle \phi_i \rangle$. The chiral superfields A_i and B_i are supposed to be charged under an unbroken factor of the low energy gauge group, such as $SU(3)_c$, so that they are forbidden from acquiring vevs.

I add a term $\breve{W}$ to the superpotential so that it now takes the form

$$\begin{aligned} W(A, B, \Phi, \Pi) &= \hat{W}(A, B, \Phi) + \breve{W}(\Phi, \Pi), \\ \hat{W}(A, B, \Phi) &= \sum_{i,j,k} \lambda_{ijk} A_i B_j \Phi_k, \qquad \breve{W}(\Phi, \Pi) = \sum_\alpha c_\alpha(\Phi) \Pi_\alpha. \end{aligned} \tag{4.1}$$

[1] It is worth noting, however, that along certain flat directions in explicit string constructions, effective mass operators for light fields are forbidden by selection rules at all nonrenormalizable orders of the superpotential, as has been emphasized recently in [11].

[2] I thank Emilian Dudas for pointing this out to us.

Here, Π_α are hidden sector matter condensate superfields. The functional $c_\alpha(\Phi)$ is left unspecified at this point. We implement dynamical supersymmetry breaking through a Veneziano-Yankielowicz-Taylor effective Lagrangian [12], following Binétruy, Gaillard and Wu (BGW) [13]:

$$\mathcal{L}_{\rm VYT} = \int \frac{E}{8R} U \left[b' \ln(e^{K/2}U) + \sum_\alpha b^\alpha \ln \Pi^\alpha \right] + {\rm h.c.}$$

The chiral superfield U corresponds to the condensing gaugino bilinear and the coefficients b' and b^α are determined by anomaly matching. We have no compactification moduli appearing, no threshold corrections, and the only GS term is the one required to cancel the $U(1)_X$ anomaly. Following the BGW formulation one obtains for the scalar potential

$$\begin{aligned} V &= \frac{1}{2}\left(\frac{2\ell}{1+f(\ell)}\right)\sum_a D_a D_a + (\ell g'(\ell) - 2)\left|\frac{b'u}{4} - e^{K/2}W\right|^2 \\ &+ \left| e^{K/2}(W_I + W G_I) - \frac{b'u}{4}G_I \right|^2 \\ &+ \left(\frac{1+\ell g'(\ell)}{16\ell^2}\right)\left[(1+2\ell b')|u|^2 - \ell e^{K/2}(W\bar{u} + \bar{W}u)\right]. \end{aligned}$$

Here, $\ell = L|, u = U|$ and the functional $f(\ell)$ is closely related to the nonperturbative correction $g(\ell)$ to the dilaton Kähler potential. Nevermind all the details in V; the point is that in principle we can find the stable vacua. What remains is purely a technical challenge.

We restrict our attention to the case where D_X is the only nonvanishing D-term, and $\langle A_i \rangle = \langle B_i \rangle = 0$. In this case $\langle V \rangle$ is, after straightforward manipulations, given by

$$\begin{aligned} V &= \frac{1}{2} g_H^2 D_X^2 + \hat{V}, \\ \hat{V} &= e^K \sigma^2 \left[b_c^2 (v^2 - 2 + \ell g') + \left(\frac{1+\ell g'}{2\ell^2}\right)(2 + 3\ell b' + \ell b_c)\right], \end{aligned} \tag{4.2}$$

where all quantities from here on out are taken at their vevs and

$$\begin{aligned} v &= \left[\sum_i |v_i|^2\right]^{1/2}, \qquad g_H^2 = \frac{2\ell}{1+f(\ell)}, \\ K &= k(\ell) + v^2, \qquad b_c = b' + \sum_\alpha b^\alpha, \\ D_X &= \sum_i q_i |v_i|^2 + \xi, \qquad \sigma = \frac{b'}{4} e^{-K/2}|u|, \\ \sigma &= \frac{1}{4}\exp\left[-\frac{1}{b_c g_H^2} - \frac{b'}{b_c}\right]\prod_\alpha \left|\frac{4 c_\alpha(v)}{b^\alpha}\right|^{b^\alpha / b_c}. \end{aligned}$$

Note that σ is essentially a reparameterization of the gaugino condensate; i.e., it is the order parameter for supersymmetry breaking. From these expressions it is not hard to work out $V_i = \partial V / \partial v_i$:

$$V_i = \bar{v}_i \left[\mathcal{A} q_i + \mathcal{B} \right] + \mu_i \hat{V}, \tag{4.3}$$

where

$$\mathcal{A} \equiv g_H^2 D_X, \qquad \mathcal{B} \equiv b_c^2 \sigma^2 e^K + \hat{V}, \qquad \mu_i \equiv \sum_\alpha \frac{b^\alpha}{b_c} \frac{\partial}{\partial v_i} \ln c_\alpha(v). \tag{4.4}$$

Note that the term $\mu_i \hat{V}$ in (4.3) was not present in our previous work. Then for the nonvanishing vevs we had only 1 constraint: $\mathcal{A} q_i + \mathcal{B} = 0$ for the minimum charge $q_i = -q$. However we now have the term $\mu_i \hat{V}$ due to the coupling $\breve{W}(\phi, \pi)$ and consequently n constraints on the n fields getting vevs. Thus we expect that (4.3) provides the necessary constraints to lift the D-moduli flat directions, barring flavor symmetries which might lead to redundant equations. In addition to the vanishing of (4.3), we also impose $V = 0$. Analysis of these two conditions, keeping in mind $\sigma^2 \ll |\xi|$, leads to the results:

$$\begin{aligned} |v_i| \quad &\sim \quad \begin{cases} \Lambda_X = \sqrt{|\xi|} & q_i = -q, \\ \sigma \qquad q_i \neq -q; & \end{cases} \\ q \quad &= \quad -\min\{q_i\}. \end{aligned} \tag{4.5}$$

Thus we get a *considerable* vacuum selection: only fields with $q_i = -q$ can get large vevs; the remainder get vevs of order the supersymmetry breaking scale. In many cases this may be sufficient to *rule out* flat directions[3] which were assumed for phenomenological reasons. The pleasing feature of this result is that it does not require a detailed knowledge of the form of $\breve{W}(\phi, \pi)$.

Note that we have only considered the case with X-Higgses charged solely under $U(1)_X$. The analogous vacuum selection which occurs for the more general case of X-Higgses charged under several factors of the gauge group will be considered elsewhere [8].

Notice that all of the quantities in (4.3) are real except $\bar{v}_i$ and μ_i. From (4.3) we see that the phase of v_i will be related to the phase of μ_i. More precisely,

$$\arg v_i = -\arg \mu_i \bmod \pi.$$

We next suppose in (4.1)

$$c_\alpha(v) = \sum_A c_{\alpha A}(v), \qquad c_{\alpha A}(v) = \lambda_{\alpha A} \prod_i (v_i)^{p_{iA}^\alpha}. \tag{4.6}$$

Then it is easy to check that (4.4) yields

$$v_i \mu_i = \sum_\alpha \frac{b^\alpha}{b_c} \frac{\sum_A p_{iA}^\alpha c_{\alpha A}(v)}{\sum_A c_{\alpha A}(v)}.$$

[3] More precisely, directions which were flat in the absence of supersymmetry breaking.

Consequently we can rewrite the minimization constraint which follows from (4.3) as

$$0 = |v_i|^2 \left[\mathcal{A} q_i + \mathcal{B}\right] + \hat{V} \sum_\alpha \frac{b^\alpha}{b_c} \frac{\sum_A p^\alpha_{iA} c_{\alpha A}(v)}{\sum_A c_{\alpha A}(v)}. \tag{4.7}$$

In the case where the sum in (4.6) has only a single term, the $c_{\alpha A}(v)$ cancel in (4.7) and no phase constraints exist. Thus, a non-monomial polynomial assumption for $c_\alpha(v)$ is required for phase stabilization.

5 A simple example

As an example, we review the simple case considered previously in [14], the case of only two fields ϕ^1, ϕ^2 of charges $q_1 = q_2 \equiv -q$ and a single matter condensate field π with superpotential coupling

$$\breve{W}(\phi, \pi) = c(\phi)\pi, \qquad c(\phi) = \lambda_1 \phi_1 + \lambda_2 \phi_2.$$

We define

$$v_1 = e^{i\varphi_1} v \cos\eta, \qquad v_2 = e^{i\varphi_2} v \sin\eta.$$

$\varphi_1 - \varphi_2$ is the phase we would like to stabilize and η is the mixing angle to the mass eigenstate basis which we would also like to stabilize. These are the D-moduli. The scalar modes corresponding to v and $\varphi_1 + \varphi_2$ are eaten by the $U(1)_X$ vector multiplet.

It is not hard to check that (4.3) gives

$$0 = b_c(\lambda_1 |v_1|^2 + \lambda_2 \bar{v}_1 v_2)(\mathcal{B} - q\mathcal{A}) + b^\alpha \lambda_1 \hat{V},$$

and a similar equation with $1 \leftrightarrow 2$, and then 2 conjugate equations. Manipulations on these four equations lead simply to

$$\frac{v_1 \bar{v}_2}{\bar{v}_1 v_2} = \frac{\bar{\lambda}_1 \lambda_2}{\lambda_1 \bar{\lambda}_2} \Rightarrow \varphi_1 - \varphi_2 = \arg(\frac{\lambda_2}{\lambda_1}) \bmod \pi.$$

It is also straightforward to check

$$\sin^2\eta = \frac{b^\alpha \hat{V}(|\lambda_1|^2 - |\lambda_2|^2) + b_c v^2 |\lambda_1|^2 (\mathcal{B} - q\mathcal{A})}{2 b_c v^2 |\lambda_2|^2 (\mathcal{B} - q\mathcal{A})}.$$

Thus the D-moduli are stabilized and the phase and mixing are determined.

6 A more realistic example

The model described here is essentially the one described in [15], supplemented by details required to stabilize the D-moduli. We write the low energy quark mass superpotential as

$$W_{\text{quark mass}} = \lambda^u_{jm} H_u Q_{jL} u^c_{mL} + \lambda^d_{jm} H_d Q_{jL} d^c_{mL}$$

with effective Yukawa matrices given by (summation indices run from 1 to 3 here and below)

$$\begin{aligned}
\lambda^u_{jm} &= \lambda_0(\delta_{j2}X^1_{3m}+\delta_{j3}X^1_{2m})+\lambda_1\lambda_{i_1i_2i_3\ell_1\ell_21j}\langle Y_1^{\ell_1i_1}Y_1^{\ell_2i_2}\rangle X^2_{i_3m}, \\
\lambda^d_{jm} &= \lambda_2\lambda_{i_1i_2i_3i_43m\ell_1\ell_2\ell_3\ell_4j}\langle Y_1^{\ell_1i_1}Y_2^{\ell_2i_2}Y_3^{\ell_3i_3}Y_3^{\ell_4i_4}\rangle.
\end{aligned}$$

The matrices X^1 and X^2 which appear represent mixing of color triplets present at the string scale to form the low energy "right-handed" quarks. The fields $Y_n^{\ell i}$ are X-Higgses evaluated at their vevs. Also present are X-Higgses denoted S^i. The quantities λ_i $(i=0,1,2)$ are phenomenological constants.

We assume that the condensing group and matter condensates in the hidden sector are such that the leading couplings are

$$\breve{W}(Y,\Pi) = \sum_{\alpha=1}^{3} c_\alpha(S,Y)\Pi_\alpha, \qquad c_\alpha(S,Y)=\lambda^\alpha_{i_1i_2i_3\ell_1\ell_2}S^{i_1}Y_1^{\ell_1i_2}Y_3^{\ell_2i_3},$$

with Π_α consisting of elementary fields such that orbifold selection rules require $\alpha\neq\ell_1\neq\ell_2\neq\alpha$ for $\lambda^\alpha_{i_1i_2i_3\ell_1\ell_2}$ to not vanish. That is, we impose string-inspired discrete symmetries on the couplings.

We assume that (q_n is the charge of the fields $Y_n^{\ell i}$ and q_S is the charge of the fields S^i):

$$\mathcal{A}q_S+\mathcal{B}\neq 0, \quad \mathcal{A}q_1+\mathcal{B}\neq 0, \quad \mathcal{A}q_3+\mathcal{B}\neq 0.$$

We make simplifying assumptions:

$$Y_1^{\ell 1}=Y_1^{\ell 2}=Y_3^{\ell 1}=Y_3^{\ell 2}=0,$$

and suppose that none of the fields $Y_1^{\ell 3}, Y_3^{\ell 3}$ vanish. Furthermore I set $T^1=T^2=T^3$ so that the only relevant string moduli dependent couplings in c_α have universal values λ. I will skip all the details and state the results which follow from setting (4.2) and (4.3) equal to zero.

First, we have that

$$S^1=S^2=0, \qquad |S^3|^2=\frac{(b'-b_c)\hat{V}}{b_c(\mathcal{A}q_S+\mathcal{B})}.$$

Second, we have the constraints (together with conjugates):

$$\begin{aligned}
0 &= b_c(\mathcal{A}q_1+\mathcal{B})w^jw^k|Y_1^{\alpha 3}|^2+b^\alpha\hat{V}(w^jY_1^{\alpha 3}Y_3^{j3}+w^kY_1^{\alpha 3}Y_3^{k3}), \\
0 &= b_c(\mathcal{A}q_3+\mathcal{B})w^jw^k|Y_3^{\alpha 3}|^2+b^\alpha\hat{V}(w^jY_3^{\alpha 3}Y_1^{j3}+w^kY_3^{\alpha 3}Y_1^{k3}),
\end{aligned}$$

where

$$w^\alpha=Y_1^{j3}Y_3^{k3}+Y_1^{k3}Y_3^{j3},$$

and $\alpha\neq j\neq k\neq\alpha$ everywhere here and below. By judiciously combining these equations it is not difficult to show that

$$0=b_c(\mathcal{A}q_1+\mathcal{B})|Y_1^{\alpha 3}|^2+b_c(\mathcal{A}q_3+\mathcal{B})|Y_3^{\alpha 3}|^2+2b^\alpha\hat{V}.$$

The 3 scale moduli η_α which parameterize solutions to this equation are defined by

$$\begin{aligned} b_c(\mathcal{A}q_1+\mathcal{B})|Y_1^{\alpha 3}|^2 &\equiv -2b^\alpha \hat{V}\cos^2\eta_\alpha, \\ b_c(\mathcal{A}q_3+\mathcal{B})|Y_3^{\alpha 3}|^2 &\equiv -2b^\alpha \hat{V}\sin^2\eta_\alpha. \end{aligned} \tag{6.1}$$

Note that η_α correspond to 3 real scalars which remain *massless* even after supersymmetry breaking and the superpotential interactions c_α are included. In order for (6.1) to have a solution, at least one of the two conditions

$$\frac{b_c(\mathcal{A}q_1+\mathcal{B})}{b^\alpha \hat{V}} < 0, \qquad \frac{b_c(\mathcal{A}q_3+\mathcal{B})}{b^\alpha \hat{V}} < 0,$$

must be satisfied. If both are satisfied the angle η_α is real; otherwise we should replace $\eta_\alpha \to \pm i\eta_\alpha$. In the model studied here $q_S = \min\{q_i\}$ so according to (4.5) it is only the field S^3 which gets a large vev and mostly cancels the FI term. On the other hand, $q_S = -2q_1 = -2q_3$. Then it is easy to show that minimization subject to cancellation of the cosmological constant yields

$$\mathcal{A}q_S+\mathcal{B} = \mathcal{O}(\sigma^4), \quad \mathcal{A}q_1+\mathcal{B} = \mathcal{A}q_3+\mathcal{B} = \frac{3}{2}\mathcal{B}+\mathcal{O}(\sigma^4).$$

Noting $\mathcal{B} = \mathcal{O}(\sigma^2)$ and $\hat{V} = \mathcal{O}(\sigma^4)$ we have from (6.1)

$$|Y_1^{\alpha 3}|^2 = \frac{4b^\alpha|\hat{V}|}{3b_c\mathcal{B}}\cos^2\eta_\alpha + \mathcal{O}(\sigma^4) = \mathcal{O}(\sigma^2),$$

$$|Y_3^{\alpha 3}|^2 = \frac{4b^\alpha|\hat{V}|}{3b_c\mathcal{B}}\sin^2\eta_\alpha + \mathcal{O}(\sigma^4) = \mathcal{O}(\sigma^2),$$

in agreement with (4.5). After some manipulation the constraints on complex phases are found to be

$$\begin{aligned} 0 \;=\; & 2|Y_1^{\alpha 3}|^2|Y_3^{j3}||Y_3^{k3}|\sin^2\eta_\alpha + 2|Y_3^{\alpha 3}|^2|Y_1^{j3}||Y_1^{k3}|\cos^2\eta_\alpha \cdot e^{i\beta_3^\alpha} \\ & +2(2-\cos^2\eta_\alpha)|Y_1^{\alpha 3}||Y_3^{\alpha 3}|\left(|Y_1^{j3}||Y_3^{k3}|e^{i\beta_2^\alpha} + |Y_3^{j3}||Y_1^{k3}|e^{i\beta_4^\alpha}\right) \end{aligned}$$

where

$$\beta_3^\alpha = 2(\phi_3^\alpha - \phi_1^\alpha) - (\phi_3^j - \phi_1^j) - (\phi_3^j - \phi_1^j),$$

$$\beta_2^\alpha = (\phi_3^\alpha - \phi_1^\alpha) - (\phi_3^j - \phi_1^j), \qquad \beta_4^\alpha = (\phi_3^\alpha - \phi_1^\alpha) - (\phi_3^k - \phi_1^k),$$

and $\phi_n^i = \arg Y_n^{i3}$. These 3 constraints on the 3 independent phases $(\phi_3^i - \phi_1^i)$ fix these pseudoscalar D-moduli. On the other hand, we have 3 orthogonal phases $(\phi_3^i + \phi_1^i)$ which do not get fixed, corresponding to 3 massless pseudoscalar moduli. The phase of S^3 also was not fixed by the minimization conditions. One linear combination of these 4 pseudoscalar moduli is eaten by the $U(1)_X$ vector boson when it becomes massive. Thus, we are left with 3 pseudoscalar D-moduli which remain massless after taking into account supersymmetry breaking and the superpotential interactions arising from $c_\alpha(S, Y)$. The "uneaten" pseudoscalar moduli pair up with the 3 real massless scalars corresponding to η_α to give three complex massless scalars.

7 Conclusions

Presumably, loop effects and additional terms added to the superpotential would stabilize these remaining moduli. Recall that I made a number of simplifying assumptions, setting several fields to zero. Already the analysis is tedious. To study the vacuum generally would be rather involved and does not make much sense to do for a toy model. In semi-realistic cases the litany of nonrenormalizable superpotential interactions which might play a significant role in D-moduli stabilization poses a technical challenge for understanding the structure of the vacuum. It is impossible to perform a systematic numerical scan of the parameter space spanned by $\mathcal{O}(50)$ independent vevs. However, such technical challenges have been overcome in other subfields of physics, such as nuclear, atomic and lattice gauge, through *semi-analytic* techniques and *importance sampling.* For instance, a Metropolis algorithm which minimizes V, or other advanced techniques for minimization of a nonlinear function of many variables, may give us a handle on global minima. Local minima identified by such techniques may represent metastable vacua with interesting cosmological consequences. Once minima are identified numerically, one could perhaps expand about these minima analytically and check for moduli; i.e., remaining flat directions.

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